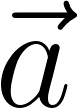
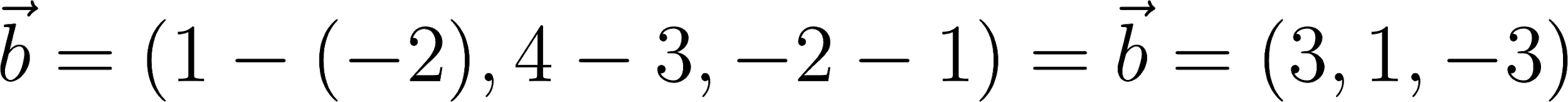
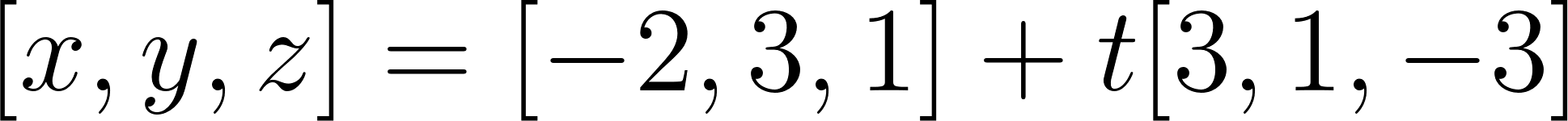
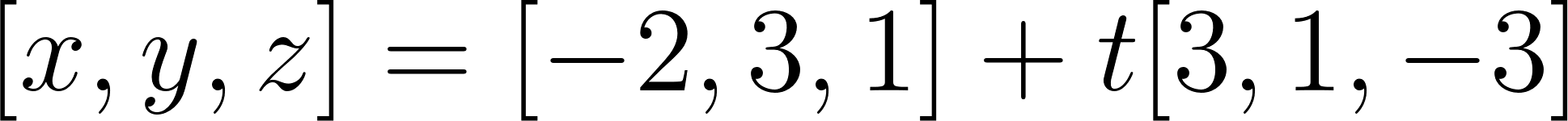
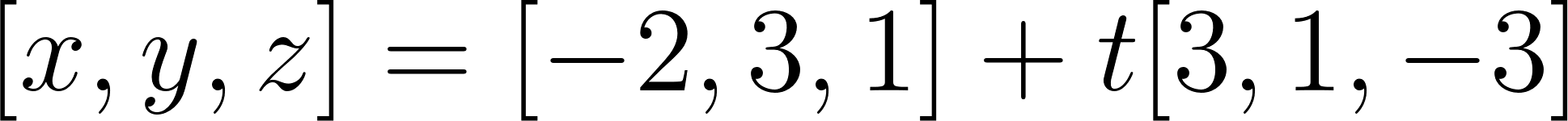
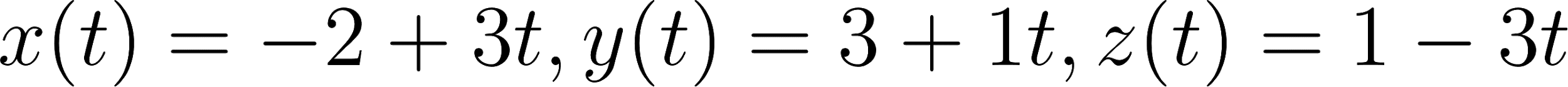
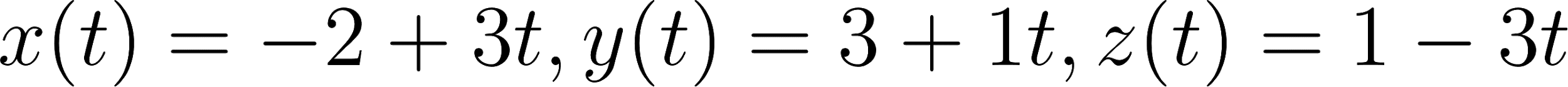
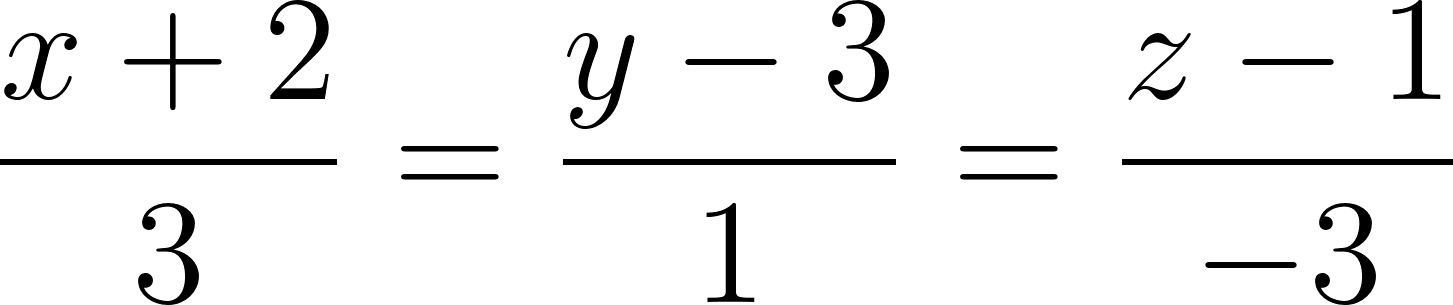
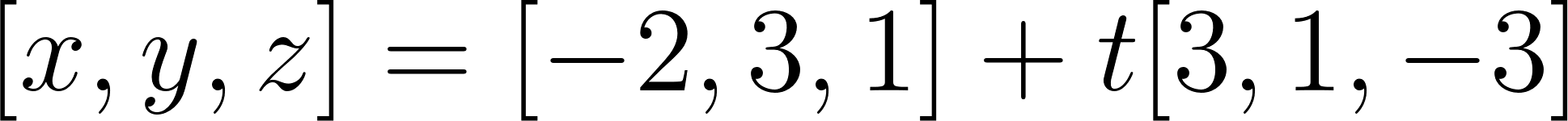
**MCV4U Intersections Unit Assignment**

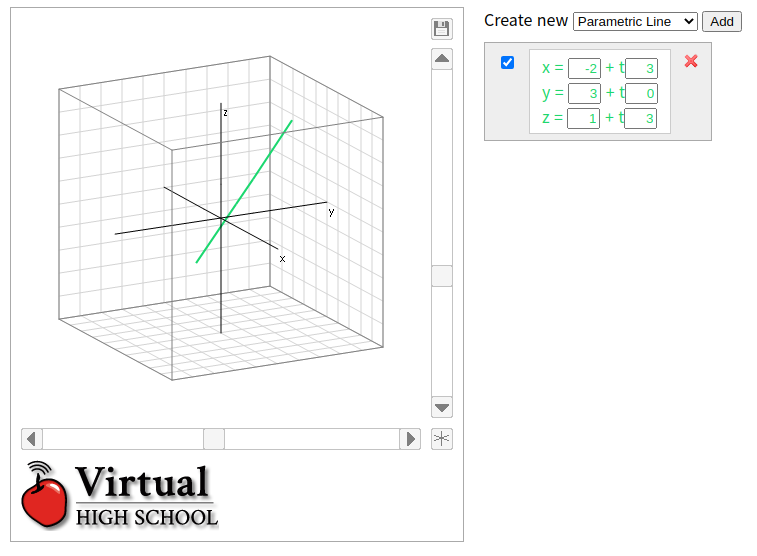
**Jin Hyung Park**

1. **The equation of a line can be determined using two points on the line.**
2. Find the vector, parametric and symmetric equations of the line through the points (–2, 3, 1) and (1, 4, –2).

* Vector Equation:
  + The point vector is [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Ba%7D#0)
  + The direction vector is [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bb%7D%3D(1-(-2)%2C4-3%2C-2-1)%3D%5Cvec%7Bb%7D%3D(3%2C1%2C-3)#0)
  + Substitute  and  into  is the following.
    - [](https://www.codecogs.com/eqnedit.php?latex=%5Bx%2Cy%2Cz%5D%3D%5B-2%2C3%2C1%5D%2Bt%5B3%2C1%2C-3%5D#0)
  + Therefore, the vector equation that passes through the given points is [](https://www.codecogs.com/eqnedit.php?latex=%5Bx%2Cy%2Cz%5D%3D%5B-2%2C3%2C1%5D%2Bt%5B3%2C1%2C-3%5D#0).
* Parametric Equation:
  + Rewrite the vector equation, [](https://www.codecogs.com/eqnedit.php?latex=%5Bx%2Cy%2Cz%5D%3D%5B-2%2C3%2C1%5D%2Bt%5B3%2C1%2C-3%5D#0), as the following.
  + [](https://www.codecogs.com/eqnedit.php?latex=%5B-2%2C3%2C1%5D%2Bt%5B3%2C1%2C-3%5D%3D(-2%2C3%2C1)%2B(3t%2C1t%2C-3t)#0)
  + [](https://www.codecogs.com/eqnedit.php?latex=x(t)%3D-2%2B3t%2C%20y(t)%3D3%2B1t%2C%20z(t)%3D1-3t#0)
* Symmetric Equation:
  + [](https://www.codecogs.com/eqnedit.php?latex=x(t)%3D-2%2B3t%2C%20y(t)%3D3%2B1t%2C%20z(t)%3D1-3t#0)
  + [](https://www.codecogs.com/eqnedit.php?latex=%5Cfrac%7Bx%2B2%7D%7B3%7D%3D%5Cfrac%7By-3%7D%7B1%7D%3D%5Cfrac%7Bz-1%7D%7B-3%7D#0)

1. Explain the features of the equations of a line that is parallel to the xz plane, but does not lie on the plane, and is not parallel to any of the axes. Include a LanGraph of your line.

* Features
  + Parallel to xy plane when the given Direction Vector is
  + Not on the xy plane when the given Position Vector is
  + Not parallel to 3 axes when the given Direction vector is
* Equations
  + [](https://www.codecogs.com/eqnedit.php?latex=%5Bx%2Cy%2Cz%5D%3D%5B-2%2C3%2C1%5D%2Bt%5B3%2C1%2C-3%5D#0)

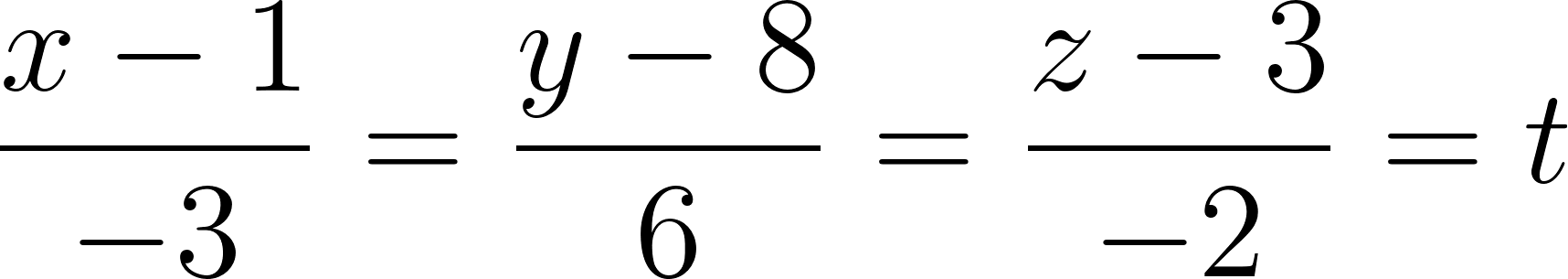


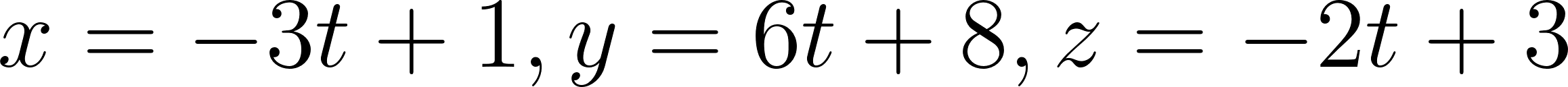
**2. Two given lines are either parallel, skew, or intersecting.**

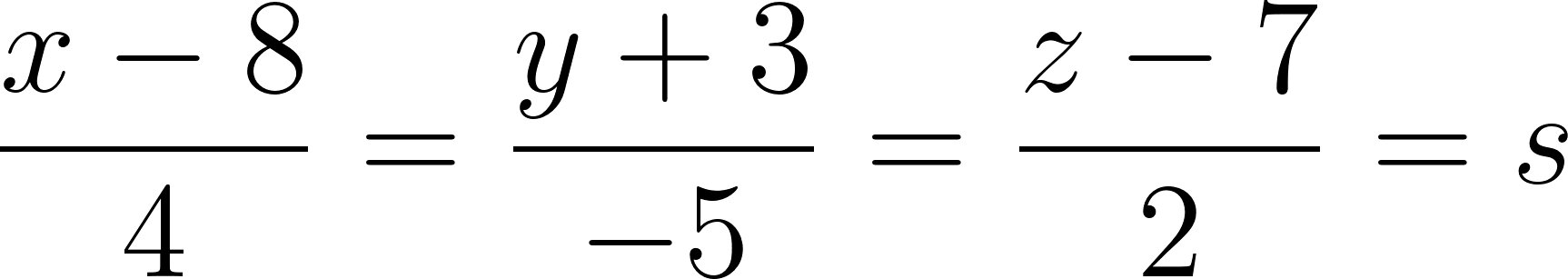
1. Determine, if there is one, the point of intersection of the lines given by the equations.

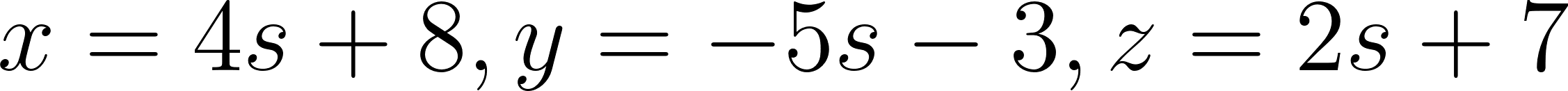


Given that, the point of intersection of the lines given by the equations, we can introduce variable t and s to get each point’s equation.

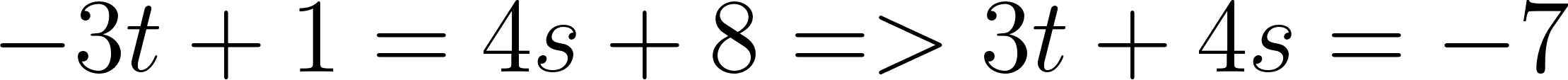
Introduce t variable: [](https://www.codecogs.com/eqnedit.php?latex=%5Cfrac%7Bx-1%7D%7B-3%7D%3D%5Cfrac%7By-8%7D%7B6%7D%3D%5Cfrac%7Bz-3%7D%7B-2%7D%3Dt#0)

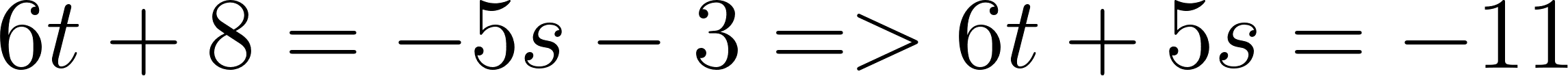
[](https://www.codecogs.com/eqnedit.php?latex=x%3D-3t%2B1%2C%20y%3D6t%2B8%2C%20z%3D-2t%2B3#0)

Introduce s variable: [](https://www.codecogs.com/eqnedit.php?latex=%5Cfrac%7Bx-8%7D%7B4%7D%3D%5Cfrac%7By%2B3%7D%7B-5%7D%3D%5Cfrac%7Bz-7%7D%7B2%7D%3Ds#0)

[](https://www.codecogs.com/eqnedit.php?latex=x%3D4s%2B8%2C%20y%3D-5s-3%2C%20z%3D2s%2B7#0)

Then, we can say that two lines will intersect.

[](https://www.codecogs.com/eqnedit.php?latex=-3t%2B1%3D4s%2B8%20%3D%3E%203t%2B4s%3D-7#0) -> (1)

[](https://www.codecogs.com/eqnedit.php?latex=6t%2B8%3D-5s-3%20%3D%3E%206t%2B5s%3D-11#0) -> (2)

Use (2), (3) to solve the variables.

->

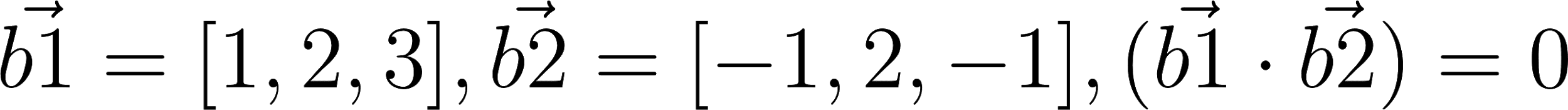
-> (2)

Substitute into (1) equation:

Thus, the point of intersection is

1. Give the equations of two lines that meet at the point (–1,5, 2) and which meet at right angles, but do not use that point in either of the equations. Explain your reasoning and include a LanGraph of your line.

The direction vectors of two perpendicular lines can be the following.

[](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bb1%7D%3D%5B1%2C2%2C3%5D%2C%20%5Cvec%7Bb2%7D%3D%5B-1%2C2%2C-1%5D%2C%20(%5Cvec%7Bb1%7D%5Ccdot%5Cvec%7Bb2%7D)%3D0#0)

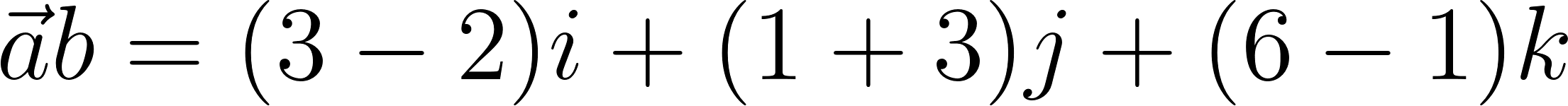
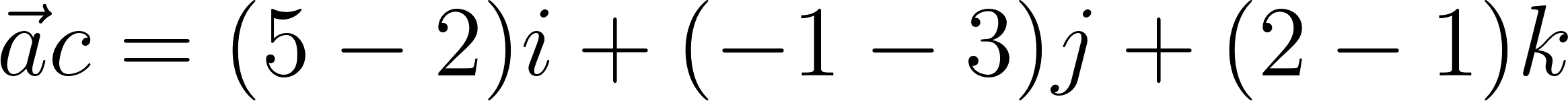
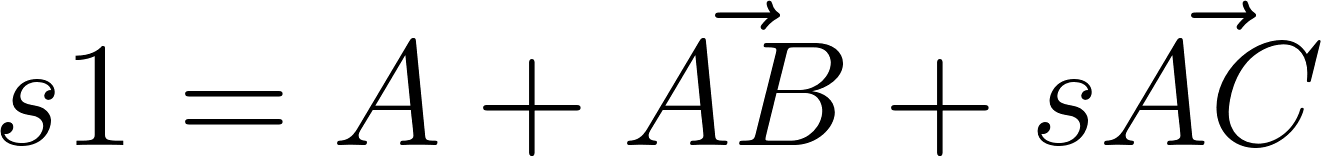
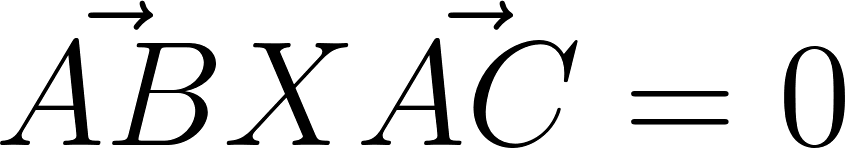
The two lines must intersect the given points at (-1,5,2). Thus, we can write the following.

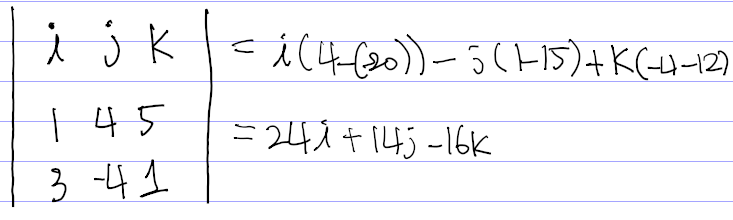
We can consider the following cases for each line to get a new position vector that intersects at (-1,5,2).

Using new position vectors,

Using new position vectors,

**3. The equation of a plane can be determined using three points on the plane.**

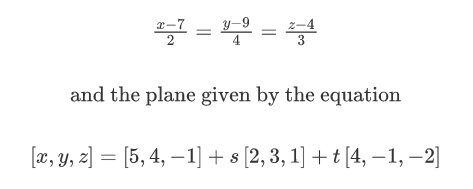
* Find the vector, parametric and general equations of the plane through the points (2, –3, 1), (3, 1, 6), and (5, –1, 2).
  + The equation of a plane can be determined using these points on the plane.
    - [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Ba%7D%7Bb%7D%3D(3-2)i%2B(1%2B3)j%2B(6-1)k#0)
    - [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Ba%7D%7Bc%7D%3D(5-2)i%2B(-1-3)j%2B(2-1)k#0)
  + The vector equation is the following.
    - [](https://www.codecogs.com/eqnedit.php?latex=s1%3DA%2B%5Cvec%7BAB%7D%2Bs%5Cvec%7BAC%7D#0)
  + The parametric equation is the following.
  + The general equation is the following.
    - [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7BAB%7DX%5Cvec%7BAC%7D%3D0#0), Two directions are perpendicular.



* Give the equation of a plane that crosses the axes at points equidistant from the origin. Explain your reasoning and include a LanGraph of your plane.
  + , ,
  + When such a plane has three points; , it crosses the three axes at the points equidistant from the origin.

**4. A line can either lie on a plane, lie parallel to it, or intersect it.**

1. Determine, if there is one, the point of intersection between the line given by the following equation and plane.



We can let formulas as the following.

Thus, we can say the following:

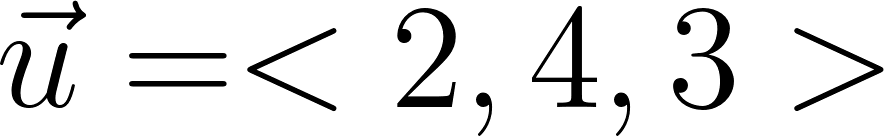
Thus, we can substitute (1) into (2) as follows.

Substitute into the formula.

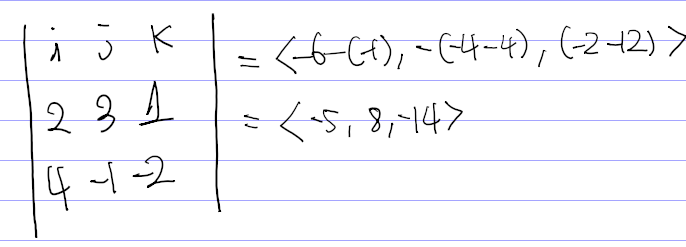
Thus, the points for the point of intersection. We can write as the following.

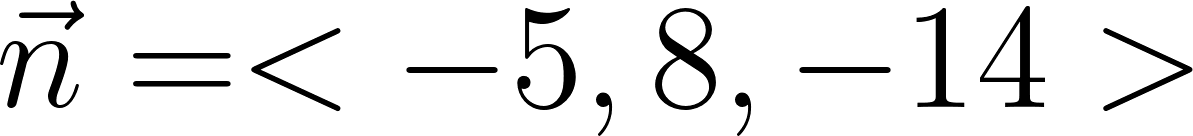
Thus, the answer is

1. Determine the angle between the line and the plane.

From the given We can say the direction vector is [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bu%7D%3D%3C2%2C4%2C3%3E#0).

We can get the normal vector by calculating the following.



Thus, [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bn%7D%3D%3C-5%2C8%2C-14%3E#0)

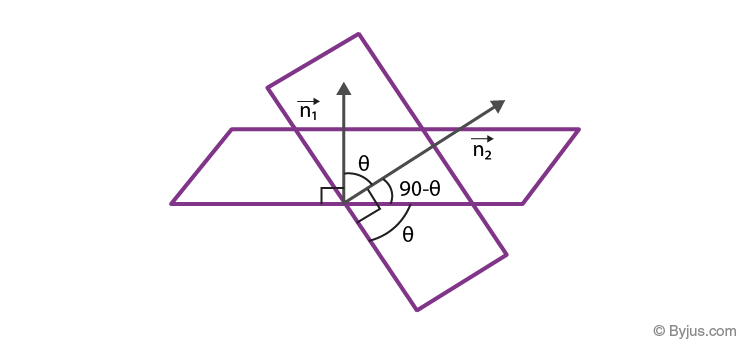
1. Give the equation of a plane and three lines, one of which is parallel to the plane, one of which lies on the plane, and one of which intersects the plane. Explain your reasoning and include a LanGraph.

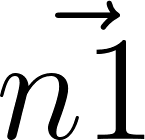
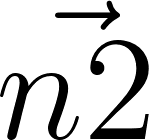
**5. The angle between two planes can also be determined.**

1. Find the angle between the planes given by the equations.

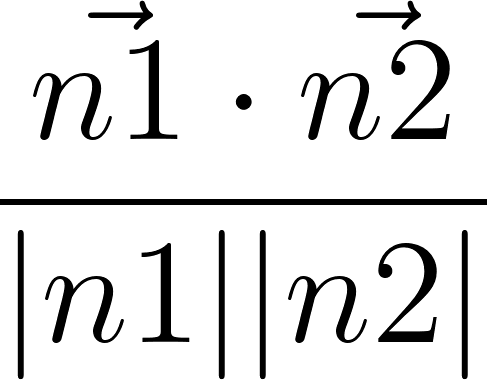
Let to be the normal vector of the plane n1.

Let to be the normal vector of the plane n2.

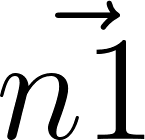
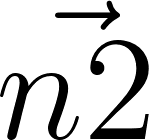
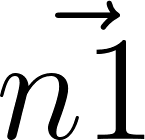


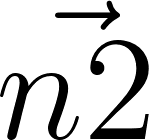
Let [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bn1%7D#0) and [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bn2%7D#0) be the two normal to the planes aligned to each other at an angle θ. From the above figure, we learnt that the angle between planes are the same as the angle between the normal vectors.

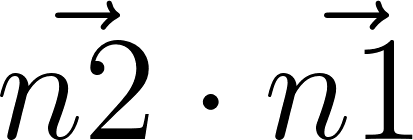
The angle between the planes are the same as the angle between the normal vectors.

[](https://www.codecogs.com/eqnedit.php?latex=%5Cfrac%7B%5Cvec%7Bn1%7D%5Ccdot%5Cvec%7Bn2%7D%7D%7B%7Cn1%7C%7Cn2%7C%7D#0)=

1. Give the equations of two planes that meet at a 90° angle. Explain your reasoning and include a LanGraph of your planes.

For the first plane, let us have [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bn1%7D#0). In this case, the normal vector of the second plane, [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bn2%7D#0), should be perpendicular to [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bn1%7D#0).

We can get the value that [](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bn2%7D#0) can be since the following.

[](https://www.codecogs.com/eqnedit.php?latex=%5Cvec%7Bn2%7D%5Ccdot%5Cvec%7Bn1%7D#0)

Two planes are normal vectors with the following.

where can be any value

where can be any value

